

# BALANCE SHEETS AND FISCAL AND EXCHANGE RATE POLICIES\*

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## ABSTRACT

We discuss the convenience of cyclical behaviour of fiscal policy in a dollarized small open economy that has to confront with adverse external shocks. We find that a countercyclical fiscal policy dominates other alternatives (procyclical or neutral fiscal policy) when fixed exchange rates are combined with financial vulnerability.

**Keywords:** *exchange rate regimes; cyclicity of fiscal policies; dollarization.*

## 1 Introduction

What are the fiscal and exchange rate optimal policies to confront with adverse external shocks has been one of the most controversial topics in the macroeconomic literature of the last decades.

According to (Erbil, 2011), optimal fiscal policy both from a keynesian or neoclassical perspective should smooth output volatility. (Lane, 2003) argues that the countercyclical fiscal policy recommendation is more clear in the keynesian case. However there exists evidence that fiscal policy has been typically procyclical in emerging economies as it is documented among others in (Kaminsky et al., 2004) and (Vegh and Vuletin, 2012).<sup>1</sup>

(Vegh and Vuletin, 2012) argue that procyclical fiscal policy can emerge as the endogenous response in a static economy with incomplete markets.

(Cuadra et al., 2010) find that procyclical fiscal policy can be optimal in an incomplete markets economy with endogenous borrowing constraints.

At the same time there has been a lot of debate about the more convenient exchange rate regime in a dollarized context. The traditional policy prescription was that flexible exchange

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<sup>1</sup>In advanced economies evidence suggests that fiscal policy has been countercyclical or acyclical.

rates facilitates macroeconomic adjustment and helps to avoid recession after a foreign shock (see for example the Mundell Fleming model). However many papers have emphasized that devaluation can have harmful consequences (via balance sheet effects) in a dollarized economy with important financial frictions, see for example (Calvo and Reinhart, 2002) or (Krugman, 1999).

(Céspedes et al., 2004) try to address this issue using an open economy model with sticky wages and without public sector. Corporate debt is dollarized and the risk premium is determined by domestic net worth as in (Bernanke and Gertler, 1989). With high indebtedness (financial vulnerability) real depreciation raises risk, and the opposite happens with financial robustness. There are negative balance sheet effects associated to a devaluation (dollarized debt) and positive effects of a devaluation (asset side). They compare the economic response to an external shock under a flexible exchange rate and under a fixed one. They conclude that flexible exchange rates dominate fixed exchange rates and play insulating role against external shocks. The intuition of such result is the following: a negative external shock requires a real devaluation, that can be reached by two ways: nominal depreciation (flexible exchange rates) or deflation (fixed exchange rates). In both cases we have negative balance sheet effects, but with fixed exchange rates (and nominal rigidity) we have an additional recessive effect. Monetary policy can do nothing against financial frictions but can undo the effects of nominal rigidity.

Another strand of the literature has stressed the possibility that with fixed exchange rates, we can obtain the same result as if we were in a flexible exchange rate economy by using fiscal policy (we can have a fiscal devaluation). An example of this literature is (Farhi et al., 2014) This literature was inspired by the situation of some countries inside the Euro area, specially Greece.

We try to analyse the issue of optimal cyclicity of fiscal policy and of optimal exchange rate regimes in a unified framework for a dollarized economy. We extend (Céspedes et al., 2004) by including a public sector that collects taxes, and issues public bonds in international markets in foreign currency. Government uses these resources to pay debt issued in previous period.

We want to address some specific issues. First, we investigate if flexible exchange rates also dominates fixed exchange rates when we include a public sector with a dollarized public debt. Another topic that we investigate is if fiscal policy can alleviate financial frictions and if it can undo the effects of nominal rigidity when monetary policy can not due to the fixed exchange rate regime. Finally we discuss if there are some contexts under which a counter cyclical fiscal policy could not be convenient.

We find that flexible exchange rates dominates even in the presence of a dollarized public debt. Another result is that fiscal policy can alleviate financial frictions. A countercyclical fiscal policy diminishes the probability of being in a financially vulnerable situation.

Our analysis suggest that countercyclical fiscal policy dominates other alternatives (procyclical or neutral fiscal policy) in the presence of fixed exchange rates. In this case a countercyclical fiscal policy diminishes the initial recession associated with an adverse financial shock, contributing to insulate the economy from financial shocks. The benefits of these kind of fiscal policy are maximized when fixed exchange rate is combined with financial vulnerability, a situation where also the posterior path of output is less negative. In a financially robust economy with a flexible exchange rate regime the convenience of a countercyclical fiscal policy is less clear, and a procyclical fiscal policy can have some benefits.

## 2 The model

We will follow closely the modellization of (Céspedes et al., 2004), and we will include a public sector.

### 2.1 Private sector

Competitive firms produce home output by a Cobb Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0 \quad (1)$$

where  $Y_t$  is home output,  $K_t$  is capital, and  $L_t$  is labor.

$L_t$  is an aggregate of heterogeneous workers:  $L_t = \left[ \int_0^1 Y_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$

The demand for worker  $i$ 's labor is given by:

$$L_{i,t} = \left( \frac{W_{i,t}}{W_t} \right)^{-\sigma} L_t \quad (2)$$

Using the first order conditions with respect to capital and labour we obtain equations 3 and 4.

$$R_t K_t = \alpha P_t Y_t \quad (3)$$

where  $R_t$  is the rental rate of capital in pesos, and  $P_t$  is the price of the home good in pesos.

$$W_t L_t = (1 - \alpha) P_t Y_t \quad (4)$$

The utility function of workers is given by:

$$\log C_{it} - \frac{\sigma - 1}{\sigma \nu} L_{it}^\nu \quad (5)$$

As a first approximation we will assume exogenous labor and we will restrict  $L_{it}^\nu$  to be one:  $L_{it}^\nu = 1$ .

Consumption ( $C_{it}$ ) is a composite of domestic production ( $C^H$ ) and imported ( $C^F$ ):

$$C_{it} = \frac{(C_{it}^H)^\gamma (C_{it}^F)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}, \quad 0 < \gamma < 1$$

The budget constraint of workers (they only consume and pay lump sum taxes) is the following:

$$W_{it} L_{it} = P_t C_{it}^H + S_t C_{it}^F + P_t T_t \quad (6)$$

Maximizing consumption subject to budget constraint we arrive to:

$$Q_t C_t = W_t L_t - T_t P_t \quad (7)$$

where price level is:

$$Q_t = P_t^\gamma S_t^{1-\gamma} \quad (8)$$

where  $\gamma$  is the domestic products expenditure (constant) share of income, and  $S_t$  is the nominal exchange rate (peso price of one dollar).

$W_{it}$  is set before observing the realization of the shocks in  $t$ , workers commit to supply labor to satisfy 2, given that we have:

$${}_{t-1}L_t^v = 1$$

In period  $t$  entrepreneurs, that are risk neutral, invest ( $K_{t+1}$ ) in domestic goods and imported production that are combined in the same way as consumption, and so the price of one capital good is  $Q_t$ . To finance their investment they use their net worth  $P_t N_t$  and they borrow abroad in dollars  $S_t D_{t+1}$ .

$$P_t N_t + S_t D_{t+1} = Q_t K_{t+1} \quad (9)$$

Given that borrowing is subject to frictions, the expected rate of return to capital (left hand side of equation 10) has to be higher than the risk free interest rate for dollars borrowed between  $t$  and  $t+1$  ( $\rho_{t+1}$ ). This safe interest rate is a random variable whose realization becomes known at  $t$ .

$$\frac{{}_t(R_{t+1}K_{t+1}/S_{t+1})}{Q_t K_{t+1}/S_t} = (1 + \rho_{t+1})(1 + \eta_{t+1}) \quad (10)$$

where  $\eta_{t+1}$  is the *corporate risk premium*, and its value, according to (Bernanke et al., 1999), is given by:

$$1 + \eta_{t+1} = F\left(\frac{Q_t K_{t+1}}{P_t N_t}\right), F(1) = 1, F'(\cdot) > 0 \quad (11)$$

Entrepreneurs collect income from capital and repay foreign debt. They consume a share of  $(1 - \delta)$  of the rest in import goods. Their net worth will be given by:

$$P_t N_t = \delta (R_t K_t - \Phi_t \alpha P_t Y_t - (1 + \rho_t) S_t D_t) \quad (12)$$

or in real terms:

$$N_t = \delta ((1 - \Phi_t) \alpha Y_t - (1 + \rho_t) E_t D_t) \quad (13)$$

## 2.2 Public Sector

In period  $t$  government collects taxes ( $P_t T_t$ ), and issues public bonds in international markets in foreign currency ( $B_{t+1}$ ). The government uses these resources to pay debt issued in the previous period ( $B_t$ ), which pays a gross nominal interest rate  $R_t^G$ , and to spend  $G_t$  on home goods. The government budget constraint is shown in equation 14.

$$\frac{S_t B_{t+1}}{(1 + \rho_{t+1})(1 + \eta_{t+1}^G)} + P_t T_t = P_t G_t + B_t S_t \quad (14)$$

For the moment we assume ( $G_t = 0$ ).

Sovereign risk follows an exogenous process:

$$\eta_{t+1}^G = \eta^G + \varepsilon_{t+1}^B \quad (15)$$

We describe three different fiscal rules in a general way by the following equation:

$$B_{t+1} = \bar{B} - \alpha^G(Z_t - \bar{Z}) \quad (16)$$

where  $Z_t$  is output in dollars ( $Z_t \equiv \frac{PY_t}{S_t}$ )

These three possible fiscal rules are the following: i) Countercyclical fiscal rule ( $\alpha^G > 0$ , herein *FRC*); ii) Procyclical fiscal rule ( $\alpha^G < 0$ , herein *FRP*); iii) Neutral fiscal rule ( $\alpha^G = 0$ , herein *FRB*).

Although the 3 fiscal rules implies a different behaviour in terms of cyclicity of fiscal policy, they all implies that public debt can not be so far from its steady state value, and ensures in this sense, the sustainability of fiscal policy. For example, under *FRC*, in face of an adverse external shock that provokes an output in dollars below its steady state value, taxes are decreased and public debt is increased but only at a distance  $\alpha^G(Z_t - \bar{Z})$  from its steady state value. If the period after the shock  $Z_{t+1}$  continues at the same distance from  $\bar{Z}$ ,  $B_{t+1}$  can not continue increasing, and so taxes have to increase in order to ensure that (see also equation 19 where it can be seen explicitly this "second period effect"). A similar reasoning applies, under *FRP*, in face of a favorable shock that generates  $Z_t > \bar{Z}$ . In appendix C we analyze which conditions have to fulfill  $\alpha^G$  to precludes an explosive dynamics of the system (and in particular of public debt).

In the *FRB* case public debt in dollars is the same each period:

$$B_{t+1} = B_t = \bar{B} \quad (17)$$

Combining equation 17 with equation 14 we arrive to the following condition that taxes have to fulfill each period to ensure that public debt in dollars remains constant.

$$P_t T_t = \frac{\bar{B} S_t i_{t+1}^G}{(1 + i_{t+1}^G)} \quad (18)$$

where  $(1 + i_{t+1}^G) = (1 + \rho_{t+1})(1 + \eta_{t+1}^G)$

If we do not restrict  $\alpha^G = 0$  (i.e. if we are under *FRC* or *FRP*), then taxes evolves each period according to:

$$P_t T_t = \frac{\bar{B} S_t i_{t+1}^G}{(1 + i_{t+1}^G)} + \alpha^G S_t \left[ \frac{Z_t - \bar{Z}}{(1 + i_{t+1}^G)} - (Z_{t-1} - \bar{Z}) \right] \quad (19)$$

Dollarized public debt provides another possible rationality to fixed exchange rates. A less volatile exchange rate implies a lower volatility of public debt service, and a lower volatility of taxes. By this channel, fixed exchange rate could induce lower volatility of consumption, an issue that we will discuss in the rest of the paper.

### 3 Equilibrium

In equilibrium the domestic production market must clear. This implies,

$$P_t Y_t = \gamma Q_t (K_{t+1} + C_t) + S_t X \quad (20)$$

where  $X$  are exogenous exports (in dollars).

Log-linearizing the equilibrium equations around a non-stochastic steady state,<sup>2</sup> we can arrive to the following expression that gives the law of motion of the corporate risk premium.

$$\begin{aligned} \eta'_{t+1} - \phi \eta'_t &= \mu \left( \frac{1 - \lambda_k}{\lambda_k} \right) (y_t - e_t) + \\ &\mu \delta (1 + \rho) \Psi [(e_t - e_{t-1}) - (y_t - y_{t-1})] + \\ &\mu \frac{\lambda_b}{\lambda_k} \left[ b_t - \left( \frac{b_{t+1} - \rho'_{t+1} - \eta_{t+1}^{G'}}{1 + i^G} \right) \right] \end{aligned} \quad (21)$$

According to equation 21 the change in risk premium can be seen as the sum of three different effects. The first term in the right hand side (RHS) shows that a depreciation decreases risk through its effect in exports, whose value in pesos are increased. Given home output, the increase in the participation of exports in demand displaces investment and implies a decrease in risk. The second term in the RHS is associated with the balance sheet channel, an unexpected devaluation or an unexpected output fall, increases the value of inherited debt with respect to net worth and increases risk. Finally the third term in the RHS is associated with fiscal policy. Given home output, if  $b_t$  is higher than  $b_{t+1}/(1 + i^G)$ , this implies that taxes are increasing, and so consumption is decreasing. The decrease in consumption generates more room for investment, and with the same net worth corporate risk has to increase.<sup>3</sup>

Combining equation B.13 in appendix, with a linearized version of equation 20 we obtain the evolution of output in dollars.

$$z_{t+1} = \rho'_{t+1} + \eta'_{t+1} + \left[ z_t + \lambda_b b_t - \frac{\lambda_b}{1 + i^G} (b_{t+1} - \rho'_{t+1} - \eta_{t+1}^{G'}) \right] \frac{1}{\lambda_k} \quad (22)$$

where output in dollars is defined as:  $z_{t+1} = y_{t+1} - e_{t+1}$

Linearizing equation 16 we obtain the following equation that describes the dynamics of public debt according to the fiscal rule

$$b_{t+1} = \frac{-\alpha^G Z}{B} z_t \quad (23)$$

The dynamics of the model can be summarized by equations 21, 22 and 23. In the model  $z_t$  is a jump variable, while  $\eta'_t$  and  $b_t$  are predetermined. In appendix A we show general conditions under which the equilibrium of a system with two predetermined variables and one non-predetermined, is determinate. In appendix C we check these conditions in our model.

In the  $FR\bar{B}$  case,  $\alpha^G = 0$  and according to equation 23:  $b_t = b_{t+1} = 0$ , and the dynamics are driven by equations 21, 22 that are also simplified. Under these conditions, as it is shown in appendix C,  $z_t$  evolves according to equation 24

$$z_t = \xi_\eta \eta'_t \quad (24)$$

where the saddle path coefficient,  $\xi_\eta$  is negative, implying that situations where the risk is below the steady state are associated with situations where output in dollars is above steady state.

<sup>2</sup>See the appendix for details.

<sup>3</sup>In the short run, given the assumption of wage rigidity, output could be determined by aggregate demand. In that case, for a given investment, consumption decrease will imply less output and less net worth and so an increase in corporate risk.

In the FRC and FRP cases ( $\alpha^G \neq 0$ ) we have to consider the three equations (21, 22 and 23). Given that this system is saddle path stable,  $z_t$  can be solved as a function of  $\eta'_t$  and  $z_{t-1}$  as it is shown in equation 25.<sup>4</sup>

$$z_t = \xi_z z_{t-1} + \xi_\eta \eta'_t \quad (25)$$

According to calibration exercises,  $\xi_z$ , and  $\xi_\eta$  increases when  $\alpha^G$  increases.  $\xi_z$  starts from approximately  $-1$  at the most procyclical fiscal policy compatible with determinacy is zero in *FRB* and it is approximately 1 at the most countercyclical fiscal policy compatible with determinacy.  $\xi_\eta$  is always negative but it is a big coefficient (in absolute value) at the most procyclical fiscal policy compatible with determinacy and it is close to zero in a strong countercyclical fiscal policy.

#### 4 Flexible exchange rates and fiscal rules

The wage setting rule implies:

$${}_{t-1}l_t = 0 \quad (26)$$

and linearizing eq 4 we obtain:

$$y_t - l_t = w_t - p_t \quad (27)$$

A flexible exchange rate regime is defined as one where Central Bank allows  $s_t$  to move freely and uses monetary policy in order to target domestic output price level ( ${}_{t-1}p_t = p_t = 0$ ). Under these conditions,  $e_t = s_t - p_t = s_t$  and (Céspedes et al., 2004) show that  $l_t = w_t = p_t = 0$ .

Using equations 20, 7 and 14, and linearizing we obtain the following version of aggregate demand:

$$y_t = \lambda_k(k_{t+1} + q_t - p_t) + (1 - \lambda_k)(s_t - p_t) - \lambda_b b_t + \frac{\lambda_b}{1 + i^G}(b_{t+1} - \rho'_{t+1} - \eta^G_{t+1}) \quad (28)$$

Linearizing the definition of the price level we arrive to:

$$q_t - p_t = (1 - \gamma)e_t \quad (29)$$

The linearized version of the production function is:

$$y_t = \alpha k_t + (1 - \alpha)l_t \quad (30)$$

In period 0, assuming that we are at the steady state and so  $k_0 = 0$ , combining 26 with 27 and 30 we obtain:

$$y_0 = \left( \frac{1 - \alpha}{\alpha} \right) p_0 \quad (31)$$

that can be seen as a simple Phillips curve.

A regime of flexible exchange rates implies  $l_0 = 0$ , using  $k_0 = 0$  (we start at the steady state), and 30, we have:  $y_0 = 0$ . Combining this with 29 and 28 we arrive to the following version of the IS:

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<sup>4</sup>See appendix C for details.

$$e_0 = \frac{-\lambda_k(1+i^G)B}{(1-\lambda_k\gamma)(1+i^G)B + \lambda_b\alpha^G Z} k_1 + \frac{\lambda_b B}{(1-\lambda_k\gamma)(1+i^G)B + \lambda_b\alpha^G Z} (\rho'_1 + \eta_1^{G'}) \quad (32)$$

According to equation 32 in order to ensure equilibrium in home goods market an investment increase has to be met with an appreciation of the real exchange rate. For a given home output a real exchange rate appreciation increases the dollar value of output, decreases the participation of exports in output, and reduces the relative value of investment. This allows the increase in investment. This relationship is affected by the fiscal rule. In the countercyclical case ( $\alpha^G > 0$ ), the appreciation and the resulting increase in output in dollars implies that taxes has to increase, and so consumption decreases. The decrease in  $e$  is associated with a decrease in exports and in addition with a decrease in consumption and so to keep equilibrium the investment increase requires less appreciation. In the pro-cyclical case ( $\alpha^G < 0$ ), appreciation will induce a decrease in taxes and an increase in consumption, and the needed appreciation for a given increase in investment will be higher.

In addition, a risk-free interest rate shock ( $\rho'_1$ ) or a sovereign risk shock ( $\eta_1^{G'}$ ) has to be accompanied by a real exchange rate depreciation (or an increase in investment) in order to keep domestic goods market equilibrium. The fiscal rule can magnify or diminish this effect. In the  $FR\bar{B}$  case ( $\alpha^G = 0$ ), both shocks imply an increase in taxes, (see equation 18), that decreases consumption (see equation 7). An increase in exports (via depreciation) or an investment increase can compensate the fall in consumption. In the countercyclical case ( $\alpha^G > 0$ ), taxes increase less than what is need to maintain public debt unchanged and so consumption and the need for the depreciation is also reduced. The opposite happens in the pro-cyclical case ( $\alpha^G < 0$ ).

Linearizing the interest parity condition we have:

$$y_t - (q_0 - p_0) - k_1 = \rho'_1 + \eta_1^{G'} + e_1 - e_0 \quad (33)$$

Equation 21, under flexible exchange rates becomes:

$$\eta_1^{G'} = \mu \left( \frac{-1 + \lambda_k}{\lambda_k} + \delta(1 + \rho)\psi - \frac{\alpha^G Y}{QK(1 + i^G)} \right) e_0 + \frac{\mu \lambda_b}{\lambda_k(1 + i^G)} (\rho'_1 + \eta_1^{G'}) \equiv \varepsilon_{\eta e} e_0 + \varepsilon_{\eta i^G} (\rho'_1 + \eta_1^{G'}) \quad (34)$$

where

$$\varepsilon_{\eta e} \equiv \mu \left( \frac{-1 + \lambda_k}{\lambda_k} + \delta(1 + \rho)\psi - \frac{\alpha^G Y}{QK(1 + i^G)} \right) \quad (35)$$

is the elasticity of the corporate risk premium with respect to a change in the real exchange rate;<sup>5</sup> and

$\varepsilon_{\eta i^G} \equiv \mu \frac{\lambda_b}{\lambda_k} (1 + \rho)^{-1} (1 + \eta^G)^{-1} > 0$  is the elasticity of the corporate risk premium with respect to a change in the interest rate that has to pay sovereign debt.

An increase in the interest rate that has to pay public debt (via  $\rho$  or  $\eta^G$ ) increases corporate risk.<sup>6</sup>

<sup>5</sup>In the  $FR\bar{B}$  case,  $\alpha^G = 0$  and so the last term in this expression is zero.

<sup>6</sup>The increase in the interest rate that have to pay public debt implies that taxes has to increase, and consumption has to decrease. In order to keep good market equilibrium investment has to increase and with the same net worth corporate risk increases. In the short run, given the assumption of wage rigidity, output could be determined by aggregate demand. In that case, for a given investment, consumption decrease will imply less output and less net



As in (Céspedes et al., 2004) we distinguish a situation of financial vulnerability ( $\varepsilon_{\eta e} > 0$ ) from a situation of financial robustness ( $\varepsilon_{\eta e} < 0$ ). The probability that an economy is in a financially vulnerable situation increases with  $\psi = SD/N$  (steady state corporate debt to net worth ratio) and decreases with  $\alpha^G$ . A countercyclical fiscal policy ( $\alpha^G > 0$ ) diminishes the probability of being in a vulnerable situation. Imposing  $\varepsilon_{\eta e} = 0$  in equation 35 we arrive to:

$$\alpha^{G*} = \left( \frac{-1 + \lambda_k}{\lambda_k} + \delta(1 + \rho)\psi \right) \frac{QK(1 + i^G)}{Y} \quad (36)$$

This  $\alpha^{G*}$  gives a threshold from which fiscal policy can change the financial situation given the other parameters. In particular a "large enough"  $\alpha^G$  (a "tough enough" countercyclical fiscal policy) can transform, given the other characteristics of the economy, a financially vulnerable situation in a robust one. Alternatively, a "low enough"  $\alpha^G$  (a "strong enough" procyclical fiscal policy) can convert a financially robust situation in a vulnerable one.

Combining equation 33 with equation 34 and equation 25 we arrive to:

$$e_0 = \frac{1}{\gamma - \xi_z - (1 - \xi_\eta)\varepsilon_{\eta e}} k_1 + \frac{1 + (1 - \xi_\eta)\varepsilon_{\eta i^G}}{\gamma - \xi_z - (1 - \xi_\eta)\varepsilon_{\eta e}} \rho_1' + \frac{(1 - \xi_\eta)\varepsilon_{\eta i^G}}{\gamma - \xi_z - (1 - \xi_\eta)\varepsilon_{\eta e}} \eta_1^{G'} \quad (37)$$

Equation 37 describes the equilibrium conditions in international financial markets, and it is a variant of the "BP" equation of (Céspedes et al., 2004) with the addition of fiscal policy effects. In the  $FR\bar{B}$  case, it can be shown that  $\xi_z = 0$  and  $\xi_\eta < 0$ . In that case we have two effects included in this equation, that affects the relation between depreciation and investment. Firstly, depreciation decreases the cost of investment in dollars (dollar price of domestic production is reduced with depreciation), this implies that in order to keep equilibrium in the loan market an increase of  $e_0$  has to be met with an increase in investment ( $k_1$ ). The size of this effect increases with the proportion of investment that is made in home goods ( $\gamma$ ). There is a second effect, that depends on the sign of  $\varepsilon_{\eta e}$ . In a financially robust economy, where an increase of  $e_0$  derives in a decrease of  $\eta_1$  ( $\varepsilon_{\eta e} < 0$ ) a depreciation affects positively investment, and so the positive relationship between investment and depreciation is reinforced. This situation is depicted by the left panel of figure 1, where the BP curve slopes up in  $(k_1, e_0)$  space. IS curve slopes down in the same plane. The situation is different in a financially vulnerable economy where  $\varepsilon_{\eta e} > 0$ . If this second effect is more important than the positive effect of the depreciation in the investment cost (i.e.  $\gamma < \xi_z + (1 - \xi_\eta)\varepsilon_{\eta e}$ ) then the BP curve slopes down in  $(k_1, e_0)$  space, as it is shown by the right panel of figure 1.

The relation between the international interest rate and the real exchange rate will also be affected by the sign of  $\varepsilon_{\eta e}$ . In a financially robust economy an increase in the international interest rate has to be met with a depreciation (that decreases corporate risk in this case), in order to keep equilibrium in financial markets. The opposite holds in the financially vulnerable case.

The equilibrium in the financial markets is also affected by sovereign risk. An increase in sovereign risk, that increases corporate risk as we have seen in equation 34, keeping investment constant, has to be met with a movement of the real exchange rate that decreases corporate risk (a depreciation in a financially robust economy, or an appreciation in a financially vulnerable one).

Financial markets equilibrium will also have some changes in the other two fiscal regimes that will be commented in section 4.1.2 and 4.1.3.

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worth and so an increase in corporate risk.

## 4.1 International risk-free interest rate shock

In this section we compare the effects of a transitory and unexpected risk-free interest rate shock in a flexible exchange rate regime combined with the 3 fiscal rules that we commented in section 2.2.

We start at the steady state. In  $t = 0$  the economy suffers a financial shock,  $\rho'_1$  rises unexpectedly, the next period  $\rho$  returns to steady state level and the system settles on saddle path converging to steady state. We try to see the results in terms of several key macroeconomic variables (output, consumption, exchange rate and others) both analytically and by simulating the model numerically. Following (Céspedes et al., 2004) we will focus on graphical analysis of the effects in the period of the shock on real exchange rate and investment. For that purpose we use two curves: the IS (see equation 32) and the BP (see equation 37), that we have already commented.

### 4.1.1 The neutral fiscal policy case ( $FR\bar{B}$ )

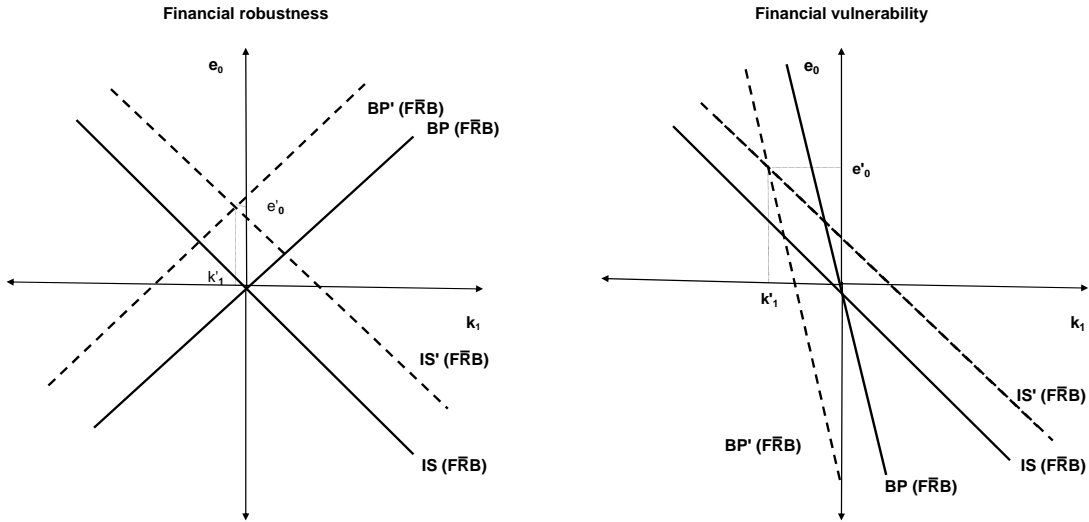


Figure 1: Financial shock in a financially robust and in a financially vulnerable economy

As we have commented in section 3, in the  $FR\bar{B}$  case  $\xi_z = 0$ , and in the saddle path  $z_t$  is only function of  $\eta'_t$  (see equation 24). Given that  $\alpha^G = 0$ , IS is simplified to:<sup>7</sup>

$$e_0 = \frac{-\lambda_k}{1 - \lambda_k \gamma} k_1 + \frac{\lambda_b}{1 - \lambda_k \gamma (1 + i^G)} (\rho'_1 + \eta_1^{G'}) \quad (38)$$

The BP is also simplified and becomes:<sup>8</sup>

$$e_0 = \frac{1}{\gamma - (1 - \xi_\eta) \varepsilon_{\eta e}} k_1 + \frac{1 + (1 - \xi_\eta) \varepsilon_{\eta i^G}}{\gamma - (1 - \xi_\eta) \varepsilon_{\eta e}} \rho'_1 + \frac{(1 - \xi_\eta) \varepsilon_{\eta i^G}}{\gamma - (1 - \xi_\eta) \varepsilon_{\eta e}} \eta_1^{G'} \quad (39)$$

If the risk-free interest rate increases ( $\rho'_1 > 0$ ), in order to maintain equilibrium in financial markets, investment has to decrease, BP has to move to the left. The increase in  $\rho$  affects directly the burden of corporate debt and indirectly via an increase in corporate risk provoked

<sup>7</sup>In the extreme case where there is no public debt ( $\lambda_b = 0$ ), the second term in the right hand side of 38 vanishes and we obtain the same IS as in (Céspedes et al., 2004).

<sup>8</sup>If public debt is zero ( $\varepsilon_{\eta i^G} = 0$ ), and equation 39 collapses to the same BP as in (Céspedes et al., 2004)

by the rise in the interest rate that has to pay public debt (see equation 34). To keep equilibrium, marginal return of capital has to increase (and for the same real exchange rate investment has to decrease). The IS curve moves to the right, because as we have commented, a risk-free interest rate shock has to be accompanied by a real exchange rate depreciation (or an increase in investment) in order to keep domestic goods market equilibrium.

These effects can be seen in figure 1 where we depicted the consequences of an unexpected increase of  $\rho_1$  ( $\rho$  is supposed to return to its steady state level in the following period). As we can see in the graph, the financial shock implies a real depreciation no matter if the economy is in a vulnerable or in a robust financial situation. Investment is reduced in the vulnerable situation, but the final result is unclear in the robust one (in the graph we assume that investment is reduced also in this case). In order to have an investment decrease after a financial shock in the robust economy the following condition must be satisfied:<sup>9</sup>

$$\lambda_b < \frac{1-\lambda_k\gamma}{\lambda_k\gamma-(1-\xi\eta)[\varepsilon_{\eta e}\lambda_k+\mu(1-\lambda_k\gamma)]} \equiv \lambda_b^*$$

Real depreciation is higher under a vulnerable situation. In this case the investment decrease is clearly more important, and, to keep equilibrium in good markets, exports (and real exchange rate) have to increase more.

The financial shock does not alter the initial product level ( $y_0$ ), given that we are in the case of flexible exchange rates  $y_0 = l_0 = 0$ . The real depreciation ( $e_0 > 0$ ) implies that product in dollars falls on impact ( $z_0 < 0$ ). On the other hand the investment decrease (that clearly happens in the vulnerable case) implies that output has to decrease in  $t = 1$  ( $y_1 = \alpha k_1 < 0$ ).

The increase in  $\rho$  and the increase in the exchange rate implies an increase in the burden of the dollarized public debt. Fiscal policy tries to keep public debt unchanged and so taxes are increased on impact (see equation 18). Consumption decreases in the period of shock due to the increase of taxes. These initial effects (increase in taxes and consumption decrease) are magnified in a financially vulnerable economy because real depreciation is higher as we commented. In the period after the shock risk free interest rate returns to equilibrium, exchange rate also tends to converge, and taxes tends to converge also to steady state level. The output decrease translates into a decrease in labor income (see equation 4) and a consumption decrease (see equation 6).

These initial effects are qualitatively similar in the financially vulnerable and in the financially robust economy (except for investment, as we have seen). After the first period, the process that the economy follows in order to return to the steady state is different in each case. The increase in  $\rho$  implies an increase in corporate risk (see equation 34). In a financially vulnerable economy these effects are reinforced because the real depreciation also leads to an increase in corporate risk (in equation 34:  $\varepsilon_{\eta e}e_0 > 0$ ). Product in dollars falls below steady state the period after the shock ( $z_1 = \xi\eta'_1 < 0$ ), after that we have a convergence process with  $z_t$ ,  $y_t$  and  $c_t$  increasing and  $\eta_t$  decreasing.

In a financially robust economy, the real depreciation leads to a fall in corporate risk premium  $\varepsilon_{\eta e}e_0 < 0$ . Corporate risk premium increases less or eventually falls. In the first case,  $z_1$  is below steady state but closer than in the vulnerable case, and so convergence is easier than in the vulnerable situation. If corporate risk premium falls, then the economy enters in the saddle

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<sup>9</sup> $\lambda_b > \lambda_b^*$  implies that public debt is very big, tax increases a lot after shock and consumption decrease is very important. In such an environment investment has to increase to keep equilibrium in the goods market and also depreciation has to be very important to increase exports and to decrease risk (depreciation reduces risk in the robust situation), compensating the increase in the risk free interest rate and the decrease in capital marginal productivity (due to capital increase).

path with an output in dollars above its steady state level ( $z_1 = \xi_\eta \eta'_1 > 0$ ), given that  $\xi_\eta < 0$ . Output in dollars is above the steady state level already one period after the shock, the economy converges fast to the steady state.

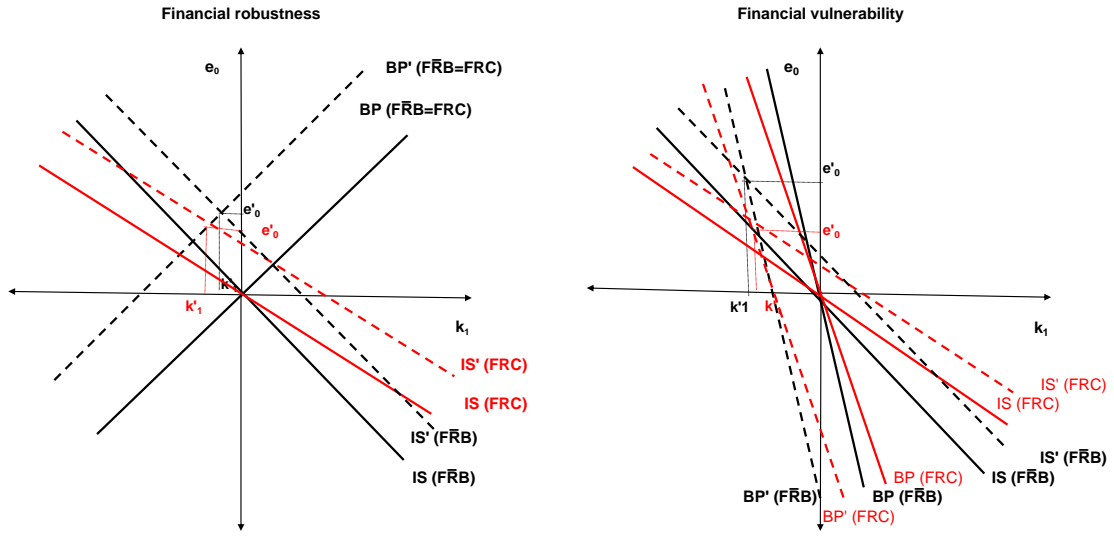


Figure 2: Financial shock.  $FR\bar{B}$  vs FRC case

#### 4.1.2 The counter cyclical fiscal policy case (FRC)

In the FRC case IS is flatter than in the  $FR\bar{B}$  situation (see equation 38). The IS moves to the right after the financial shock but in a lesser magnitude. If the slope and movement of the BP curve is the same than in the  $FR\bar{B}$  case, after a financial shock we would have less depreciation no matter if we are in financially robust or vulnerable situation. Investment decrease would be more important in the financially robust case but lower in the vulnerable case (see in figure 2 the intersection of dotted red line with the black dotted one).

In financial markets the elasticity of the corporate risk premium with respect to a change in the real exchange rate ( $\varepsilon_{\eta e}$ ) becomes more negative in the financially robust case, and less positive in the vulnerable situation (see equation 35). If these were the only change in the financial markets equilibrium, then BP would become flatter and move less after financial shock in the financially robust case, reinforcing the conclusions about depreciation (less depreciation under FRC) and eventually changing the conclusions about investment (that could decrease less than in the  $FR\bar{B}$  case). In the financially vulnerable case, this change could imply that BP becomes steeper, eventually changing the sign of the slope (and so we would be in a financially robust case again). The conclusions about depreciation and investment (less depreciation and less investment decrease than in the  $FR\bar{B}$  case), would be reinforced.

Finally as can be seen in equation 37, we also have to consider what happens with  $\xi_z$  and  $\xi_\eta$ . According to calibration exercises, under financial robustness, BP would have a similar behavior than in the  $FR\bar{B}$  case. Under financial vulnerability BP would be flatter and moves less after a financial shock than in the  $FR\bar{B}$  case.

As can be seen in figure 2, in our benchmark case, depreciation after a financial shock is lower in the FRC case with respect to  $FR\bar{B}$ . The investment decrease is higher under financial robustness and lower under financial vulnerability (for  $\alpha^G$  large enough).

Public debt service increases but lower than under  $FR\bar{B}$  (the increase in  $\rho$  is the same but there are less depreciation). In addition, given that dollar output is lower than its steady state

level, fiscal policy allows an initial increase in public debt and as a result, taxes are increased in a lesser magnitude than in the neutral fiscal policy case (if fiscal policy is strongly countercyclical, i.e.  $\alpha^G$  is large, taxes can decrease). These different behaviour of taxes implies a different situation in terms of consumption that decreases less or eventually increases in the initial period.

In the next period, as in  $FR\bar{B}$ , risk free interest rate returns to equilibrium and exchange rate also tends to converge. However public debt is higher than its steady state level, and dollar output is closer to its steady state level, so public debt has to decrease (see equation 16) and taxes has to increase. As a consequence, consumption in the period after the shock is lower under  $FRC$  than under  $FR\bar{B}$  (now we have an increase in taxes added to the effect of less output). Eventually (with  $\alpha^G$  large enough) in a vulnerable situation,  $k_1$  and  $y_1$  decrease less, but the effect of the increase of taxes prevails and  $c_1$  is lower under  $FRC$ . In a robust situation  $k_1$  and  $y_1$  decrease more, and this effect is added to the increase of taxes and so the decrease of  $c_1$  is magnified.

After these initial effects, the convergence to steady state is similar to the  $FR\bar{B}$  case. As  $\alpha^G$  increases, the periods needed to arrive to steady state increase also.

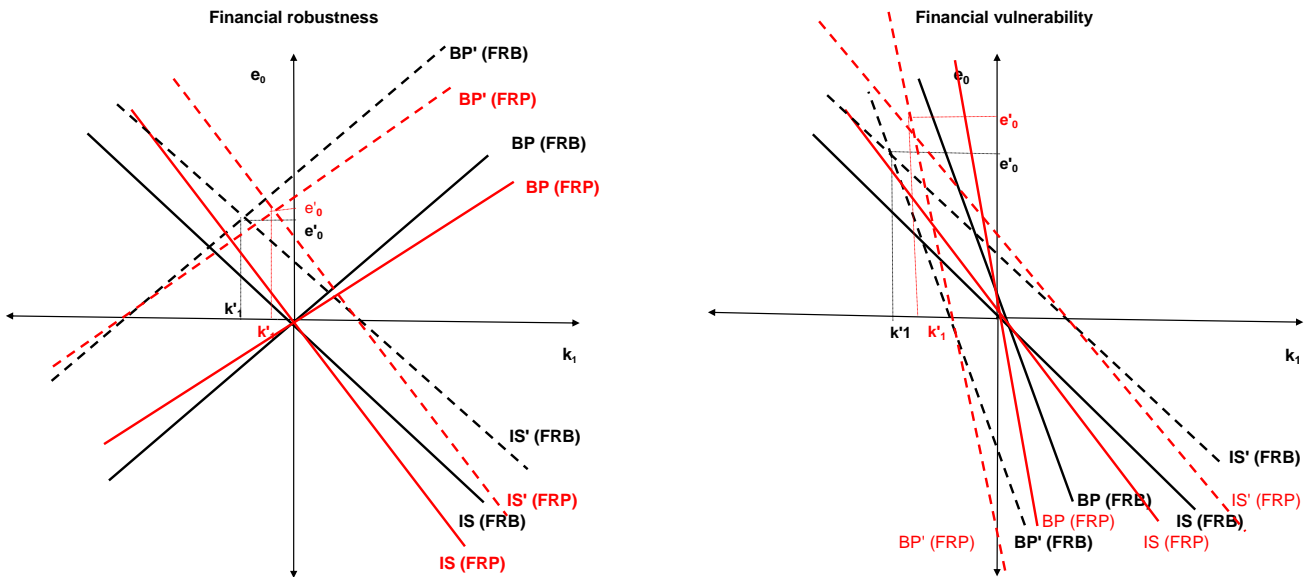


Figure 3: Financial shock.  $FR\bar{B}$  vs FRP case

#### 4.1.3 The Procyclical fiscal policy case (FRP)

In the FRP case ( $\alpha^G < 0$ ) IS is steeper than in the  $FR\bar{B}$  situation (see equation 38). IS moves to the right after the financial shock but in a greater amount. If the slope and movement of the BP curve is the same than in the  $FR\bar{B}$  case, after a financial shock we would have more depreciation no matter if we are in a financially robust or vulnerable situation. Investment decrease would be less important in the financially robust case but higher in the vulnerable case (see in figure 2 the intersection of dotted red line with the black dotted one).

In financial markets the elasticity of the corporate risk premium with respect to a change in the real exchange rate ( $\varepsilon_{\eta e}$ ) becomes less negative in the financially robust case, and more positive in the vulnerable situation (see equation 35). If these were the only change in the financial markets equilibrium, then BP would become steeper and move more after financial shock in the financially robust case, reinforcing the conclusions about a depreciation (more depreciation under FRP) and eventually changing the conclusions about investment (that could

decrease more than in the  $FR\bar{B}$  case). In the financially vulnerable case, this change could imply that BP becomes flatter, reinforcing the conclusions about depreciation and investment (more depreciation and more investment decrease than in the  $FR\bar{B}$  case).<sup>10</sup>

Finally, we also have to consider what happens with  $\xi_z$  and  $\xi_\eta$ . According to calibration exercises, under financial robustness, BP would be flatter and moves less after a financial shock than in the  $FR\bar{B}$  case. Under financial vulnerability BP would be steeper and moves more after a financial shock than in the  $FR\bar{B}$  case.

As can be seen in figure 3, in our benchmark case, depreciation after a financial shock is higher in the FRP case with respect to  $FR\bar{B}$ , while investment decrease is lower.

Under  $FRP$  public debt service initially increases more than under  $FR\bar{B}$  (the increase in  $\rho$  is the same but there are more depreciation). Furthermore, dollar output is lower than its steady state level, and fiscal policy reacts decreasing public debt and increasing taxes more than in the neutral fiscal policy case. As a result, initial consumption decrease is higher under  $FRP$ .

In the next period, public debt is below its steady state level, and dollar output is closer to its steady state level, so public debt has to increase (see equation 16) and taxes has to decrease. Consumption in period 1 is higher under  $FRP$  due to the decrease of taxes and also because  $k_1$  and  $y_1$  decrease less.

After these initial effects, the convergence to steady state is similar to the other cases. As in  $FRC$ ,  $\alpha^G$  increases (in absolute value) the periods needed to arrive to steady state increase also.

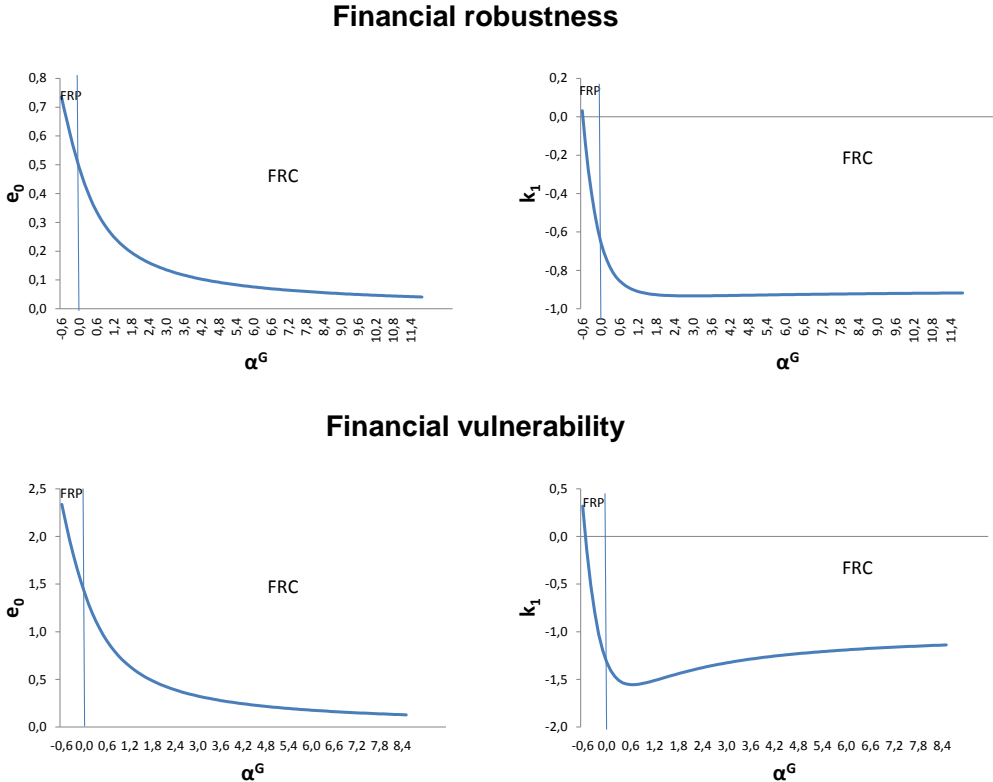


Figure 4: Effects on  $e_0$  and  $k_1$  of different values of  $\alpha^G$

<sup>10</sup>In the financially vulnerable case combined with an extreme pro-cyclical fiscal policy, we cannot discard the possibility that BP curve become flatter than IS. In that case the economy requires an appreciation of the real exchange rate and an investment increase after the financial shock. In such environment debt and financial imperfections are so important that the economy requires currency appreciation (to decrease dollar value of debt and financial risk) and an investment increase to keep the goods market equilibrium and compensate for the decrease of net exports associated with currency appreciation.

#### 4.1.4 Effects of the shock under different values of $\alpha^G$

##### *Effects on $e_0$ and $k_1$*

In figure 4 we can see the result of the shock in terms of  $e_0$  and  $k_1$  for different values of  $\alpha^G$ . To obtain this results we simulate the model numerically, following the calibration of (Céspedes et al., 2000), with the addition of fiscal parameters calibrated with a similar criteria (basically more risk and more indebtedness in public sector for a vulnerable situation). As can be seen system seems to be more stable under FRC and financial robustness (we have determinacy for more values of  $\alpha^G$  in this situation). On the contrary under a strong pro-cyclical fiscal policy the equilibrium is not determined.

The threshold ( $\alpha^{G*}$ ) that transforms vulnerability in robustness is not binding under our calibration. An extremely strong countercyclical fiscal policy ( $\alpha^G = 26.23$ ) convert the financially vulnerable environment that we simulate into a robust one, but the equilibrium is not determined for  $\alpha^G > 8.5$ . In the case of robustness, a procyclical fiscal policy with an  $\alpha^G < -0.661$  turn robustness into vulnerability. However, the system is not determined for  $\alpha^G < 0.676$ , so there is a small parameter space for which we can have a financially robust situation that is changed into a vulnerable one via fiscal policy (keeping the determinacy of the equilibrium).

We can also see that depreciation after shock decreases with  $\alpha^G$ , procyclical fiscal policy implies more depreciation and countercyclical fiscal policy promotes less depreciation. This result was the most plausible analytically as we have seen previously and seems to be a very robust one.

The maximum investment decrease is reached with a mildly countercyclical fiscal policy while the minimum decrease is obtained with a mildly procyclical fiscal policy. Under vulnerability, we also have that investment decrease is lower with a strong countercyclical fiscal policy compared with a neutral fiscal policy ( $\alpha^G > 3,2$  in our simulations).

Both depreciation and investment decrease are magnified by vulnerability. Under vulnerability the increase of  $\rho$  is accompanied by an increase in corporate risk premium and so the investment decrease is more important. More investment decrease requires more depreciation in order to keep good markets equilibrium through the increase in exports.

##### *Effects on $y_0$ and $c_0$*

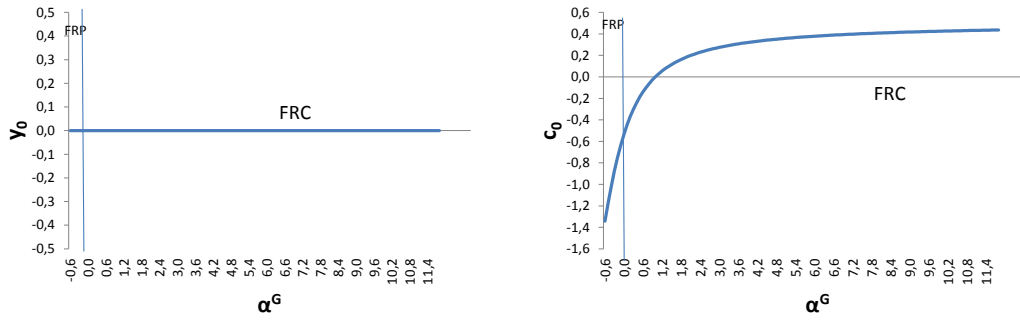
In figure 5 we can see the result of the shock in terms of  $y_0$  and  $c_0$  for different values of  $\alpha^G$ . Given that we are assuming flexible exchange rates the initial effect in output is zero for all  $\alpha^G$ .

Despite this zero initial output effect, consumption decrease on impact under neutral fiscal policy ( $\alpha^G = 0$ ) because of the increase of taxes needed to keep public debt unchanged. Under vulnerability this impact is higher because there are more depreciation and so the burden of the dollarized public debt is bigger. With  $\alpha^G > 0$  (countercyclical fiscal policy) taxes increase less and consumption also decreases less. With an  $\alpha^G$  large enough ( $\alpha^G > 1$  in robustness or  $\alpha^G > 2.3$  in vulnerability) consumption directly increase in the period of the shock. Under a procyclical fiscal policy  $\alpha^G < 0$  the effects goes in the opposite direction.

#### 4.1.5 The dynamics after the shock under different values of $\alpha^G$

In figure 6 we show the dynamics of public debt, taxes, output and consumption for 10 periods (the period of the shock and 9 more periods), under the three fiscal policy regimes that we are considering. The black solid line represents the behaviour of the variables under a neutral

### Financial robustness



### Financial vulnerability

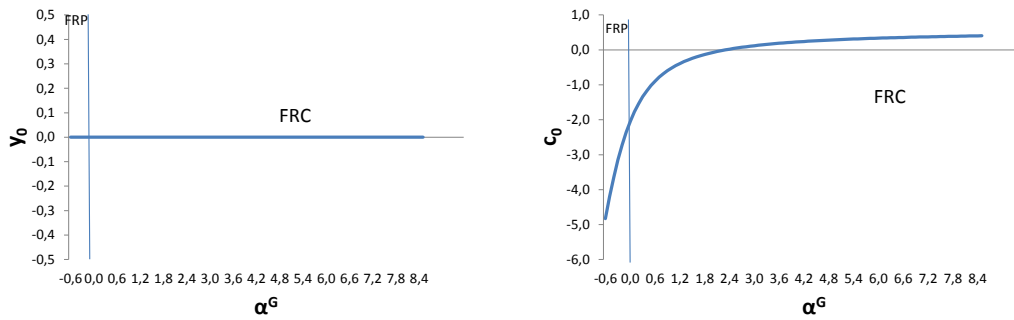


Figure 5: Effects on  $y_0$  and  $c_0$  of different values of  $\alpha^G$

fiscal policy. For a procyclical fiscal policy (red line with circles) we select  $\alpha^G = -0.4$  (if we choose a strong procyclical fiscal policy we have no determinacy). In the countercyclical case (blue dashed line) we select  $\alpha^G = 3.5$  in order to see a case where the investment decrease in a vulnerable economy is lower than under FRB (for this we need  $\alpha^G > 3.2$  with our calibration).

In the figure we can see quantitatively the effects that we have commented previously.

The period of the shock ( $t = 0$ )  $\rho$  increases unexpectedly 1% and returns to steady state level in  $t = 1$  ( $\rho$  is 4% previous and after the shock and is 5% the period of the shock).

Under FRB (black solid line) public debt in dollars ( $B$ ) remains constant and fiscal policy increases taxes in the period of the shock in order to ensure this public debt behavior. This implies an increase of taxes of about 20% of its steady state (SS) level in the financial robust economy and an increase a little lower (in % of SS) in the financial vulnerable one. Given that we are working without public expenditure taxes in SS are just what is needed to pay public debt service (see equation 18). Public debt is 4 times bigger under vulnerability and SS sovereign risk is also bigger in this situation. As a consequence taxes in SS are much higher in a vulnerable situation, and increase more (in absolute value) in order to ensure a constant public debt. This implies that consumption decreases more under vulnerability. The drop in consumption is about 2% from its SS level under vulnerability and about half percent under robustness. Output remains in its steady state level in the period of the shock, ensured by the flexible exchange rate regime. In the period after the shock ( $t=1$ ) taxes are almost in its steady state level. However consumption experiments a second drop, this time due to the drop in output level. The fall in output is motivated by the investment drop in the period of the shock and is about 0.5% of its SS level under vulnerability and half of this under robustness. Consumption is 0.3% below SS in this second period under vulnerability and again half of this under robustness. In  $t=2$  output and consumption are almost at SS under robustness but they are again below SS under vulnerability.



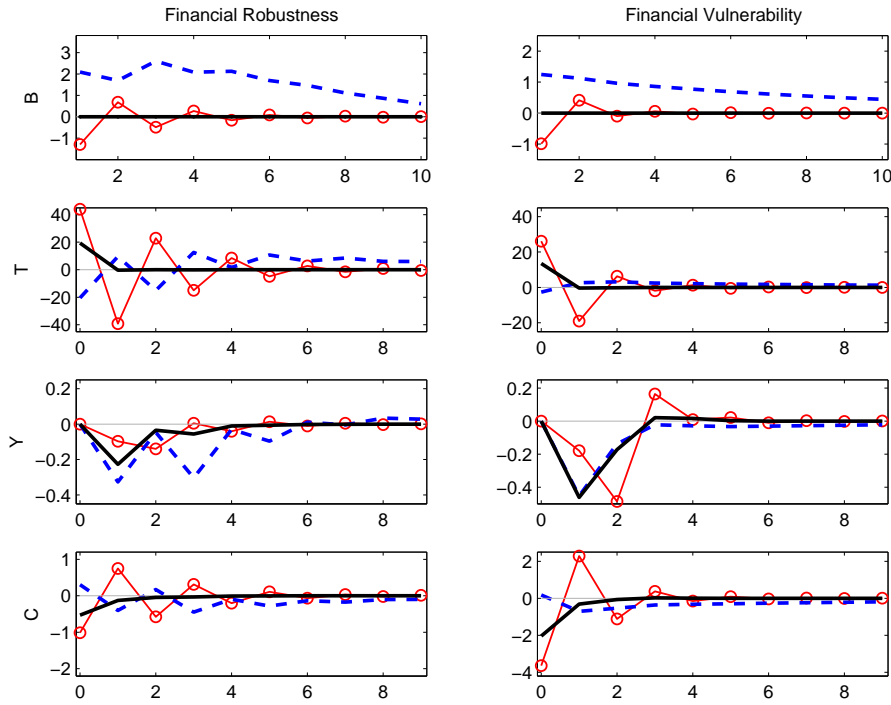


Figure 6: Impulse Responses to a financial shock in a flexible exchange rate economy under FRB (—); FRC (---) and FRP (-o-).

If fiscal policy is countercyclical (blue dashed line), public debt is allowed to increase in the period of the shock and so taxes increase less than in FRB. In the case shown in figure 6 ( $\alpha^G = 3, 5$ ), taxes directly decreases in  $t=0$ . This different movement of fiscal policy prevents consumption from decrease, and with our calibration consumptions increases a little bit. In the next period, output decreases more than with FRB under robustness and decreases a slightly less under vulnerability in the example of FRC shown in figure. As we have commented, under robustness FRC implies more investment decrease in the period of the shock and less investment decrease under vulnerability (for  $\alpha^G$  large enough), we are using an  $\alpha^G$  that is a bit higher than the threshold so we have this effect. Taxes increase in this second period because output in dollars is closer to SS and so public debt has to decrease. Consumption decrease a bit more than in FRB in the period after the shock, due to the increase of taxes, amplified by output effect on robustness. After that public debt decreases gradually and the rest of the variables converges to SS. Under robustness with this tough countercyclical fiscal policy the convergence process is oscillant and longer than with FRB. Under FRC there are less depreciation as a response to the shock. This has a beneficial effect for investment under vulnerability (decrease corporate risk and so promotes investment) but has a negative effect under robustness (corporate risk is increased by this effect). The convenience of a tough countercyclical fiscal policy are less clear for a flexible exchange rate and financially robust economy.

Under *FRP* (red line with circles) fiscal policy reacts decreasing public debt and increasing taxes more than in the neutral fiscal policy case. As a result, initial consumption decrease is higher under *FRP*. Investment decrease less with this fiscal policy and the decrease of output in the next period is lower. In  $t=1$  public debt increases and taxes are decreased. Consumption in period 1 is higher under *FRP* due to the decrease of taxes and also due to the lower decrease of output. This higher level of consumption in  $t=1$  promotes less investment in this period and in  $t=2$  output is lower under FRC. This effect is very important under vulnerability where this

drop of output has the same magnitude than the drop in  $t=1$  under FRB. As happens with FRC combined with robustness, under FRP the convergence to SS is oscilant.

## 5 Fixed exchange rates

We define a fixed exchange rate regime as one where the Central Bank fixes the nominal exchange rate  $s_t = 0$ , and so  $e_t = -p_t$ . The Phillips curve (equation 31) now changes to:

$$y_0 = - \left( \frac{1 - \alpha}{\alpha} \right) e_0 \quad (40)$$

In a similar vein as we did for flexible exchange rates we can obtain the IS. The difference is that now monetary policy does not ensure  $y_0 = 0$ , instead  $y_0$  is given by 40, and so IS will be:

$$e_0 = \frac{-\lambda_k(1+i^G)B\alpha}{(1-\lambda_k\gamma\alpha)(1+i^G)B+\lambda_b\alpha^G Z} k_1 + \frac{\lambda_b B \alpha}{(1-\lambda_k\gamma\alpha)(1+i^G)B+\lambda_b\alpha^G Z} (\rho'_1 + \eta_1^{G'}) \quad (41)$$

The law of motion of risk premium (equation 21), now becomes:

$$\eta'_1 = \alpha^{-1} \mu \left( \frac{-1 + \lambda_k}{\lambda_k} + \delta(1 - \rho) \psi - \frac{\alpha^G Y}{QK(1 + i^G)} \right) e_0 + \frac{\mu \lambda_b}{\lambda_k(1 + i^G)} (\rho'_1 + \eta_1^{G'}) \equiv \alpha^{-1} \varepsilon_{\eta_e} e_0 + \varepsilon_{\eta_i^G} (\rho'_1 + \eta_1^{G'}) \quad (42)$$

Equation 42 is the same as 34 except for the fact that now the effect of a devaluation on risk is amplified by  $\alpha^{-1}$  given that under fixed exchange rates a devaluation is associated with deflation, output and net worth decrease and so with more risk.

As in the flexible case we will distinguish a situation of financial vulnerability ( $\varepsilon_{\eta_e} > 0$ ) from a situation of financial robustness ( $\varepsilon_{\eta_e} < 0$ ).

Finally, the combination of 42 with 33 gives the BP in the fixed exchange rates case:

$$e_0 = \frac{1}{\gamma - \xi_z \alpha^{-1} - (1 - \xi_\eta) \varepsilon_{\eta_e} \alpha^{-1}} k_1 + \frac{1 + (1 - \xi_\eta) \varepsilon_{\eta_i^G}}{\gamma - \xi_z \alpha^{-1} - (1 - \xi_\eta) \varepsilon_{\eta_e} \alpha^{-1}} \rho'_1 + \frac{(1 - \xi_\eta) \varepsilon_{\eta_i^G}}{\gamma - \xi_z \alpha^{-1} - (1 - \xi_\eta) \varepsilon_{\eta_e} \alpha^{-1}} \eta_1^{G'} \quad (43)$$

### 5.1 Financial shocks under a neutral fiscal policy

Under fixed exchange rates the graphical analysis based on the IS and BP curves is essentially the same that we showed previously with the only exception that now both curves are flatter than in the flexible case. After a financial shock like the one showed in figure 1, investment falls and real exchange rate is depreciated, as happened in the flexible case.

The fall in investment is larger, and the real exchange rate depreciation is lower under fixed rates. There is also an important difference with respect to the flexible counterpart. Under fixed rates the real exchange rate depreciation is associated with price deflation and with a decrease on impact on output and employment, as can be seen in equation 40.

## 5.2 Financial shocks in the FRC and FRP case

The qualitative responses of the system to a shock under fixed exchange rates in the FRC and FRP case are well represented by figures 2 and 3.<sup>11</sup> As we have shown, there is less depreciation in the FRC case, and more depreciation in the FRP case compared with the neutral fiscal policy. This implies that the initial recession is diminished under FRC and amplified under FRP case. The benefits of the counter cyclical fiscal policy are enlarged in the financially vulnerable situation, where investment decrease is also lower (for  $\alpha^G$  large enough) and so the recession of the following period is also diminished given that the capital decrease is lower.

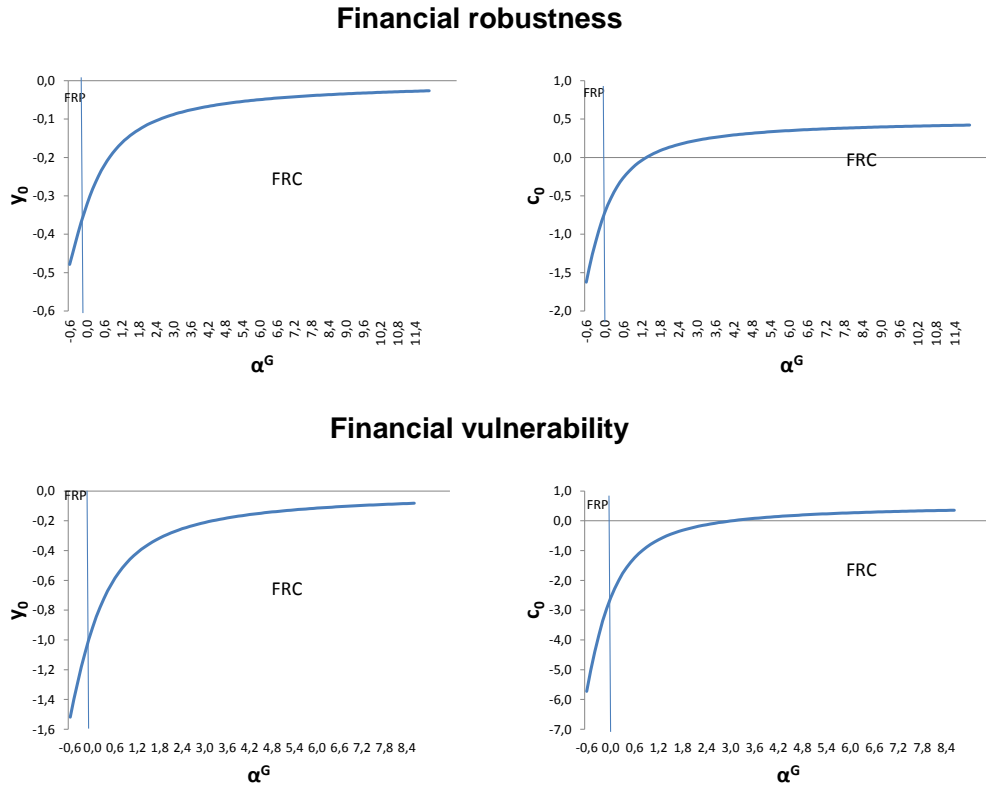


Figure 7: Effects on  $y_0$  and  $c_0$  of different values of  $\alpha^G$

### 5.2.1 Effects of the shock under different values of $\alpha^G$

In figure 7 we can see the result of the shock in terms of  $y_0$  and  $c_0$  for different values of  $\alpha^G$ . The main difference with figure 5 is that output decreases the period of the shock as a result of the nominal rigidity combined with fixed exchange rates. Initial depreciation decreases with  $\alpha^G$ , and this translates directly to output (see equation 40). The initial recession decreases with  $\alpha^G$ . For a very tough countercyclical fiscal policy the initial recession almost vanish, and we are close to replicate (via fiscal policy) the result of flexible exchange rate.

The burden of the dollarized public debt increases less under fixed exchange rates due to the lower real depreciation under this regime. This put less pressure on taxes and consequently on consumption. However for a neutral fiscal policy the consumption decrease is higher under fixed exchange rates because the output drop effect prevails. Once again the consumption decrease is

<sup>11</sup>IS and BP curves are flatter under Fixed exchange rates than in the flexible counterpart, implying less depreciation and more capital decrease following a financial shock.

more important under FRP and less important under FRC. For an  $\alpha^G$  large enough, the initial drop in consumption disappears and consumption eventually increase in the period of the shock.

### 5.2.2 The dynamics after the shock under different values of $\alpha^G$

In figure 8 we show the dynamics of public debt, taxes, output and consumption the period of the shock and the following 9 periods for FRB, FRC and FRP under fixed exchange rates. We use the same  $\alpha^G$  that we used in the flexible exchange rates case ( $\alpha^G = -0.4$  for FRP and  $\alpha^G = 3.5$  for FRC).

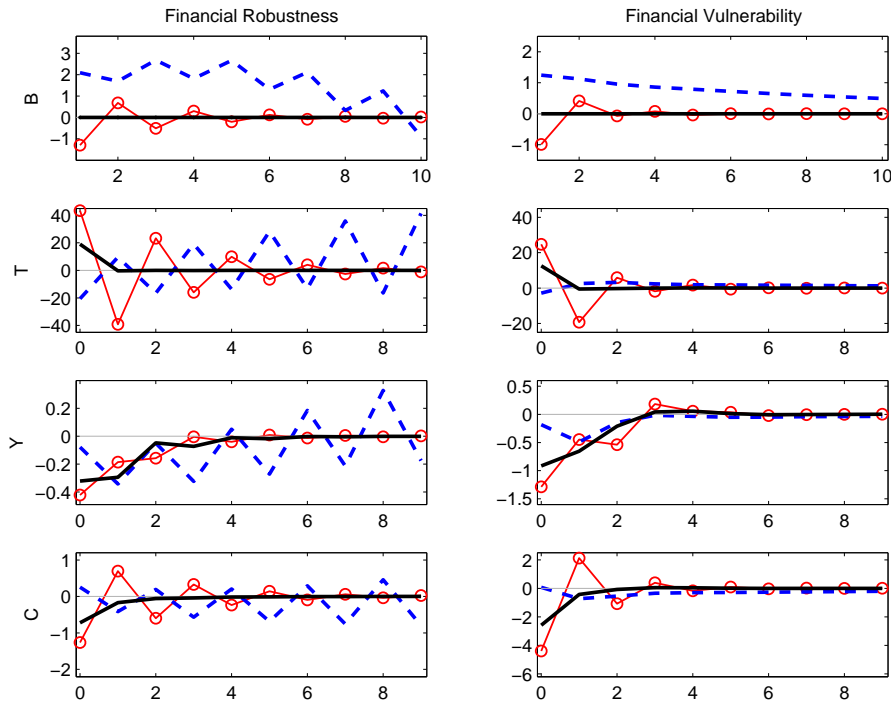


Figure 8: Impulse Responses to a financial shock in a fixed exchange rate economy under FRB (—); FRC (- - -) and FRP (-o-).

Under FRB, public debt in dollars remains constant and fiscal policy increases taxes as in flexible exchange rates. Taxes increase less than in flexible rates because we have the same increase in  $\rho$ , but now real exchange rate increases less. Indeed nominal exchange rate remains constant but  $P$  decreases, so even without considering the increase in  $\rho$ , taxes are increased in real terms. aOutput decreases in the period of the shock, about 0.3% in robustness and about 1% in a vulnerable situation. Consumption decreases more than in flexible rates as a consequence of the output drop despite the fact that taxes increase less.

In the period after the shock ( $t=1$ ) taxes are almost in its steady state level, under FRB. However consumption experiments a second drop, more intense than under flexible rates, because output is below SS (investment decreased more under fixed exchange rates). After that output and consumption converges to its steady (more slowly under vulnerability).

When fiscal policy is countercyclical, public debt increases and taxes increase less than in FRB, as in flexible rates. Now this behaviour has the effect of avoiding the drop in consumption but also (by inducing a lower depreciation) implies a drop in output less intense ( $-0.2\%$  instead

of  $-1\%$  under vulnerability and  $-0.2\%$  instead of  $-0.3\%$  under robustness). In the second period, output decreases a bit more than with FRB in robustness and decreases less under the example of FRC shown in figure. Taxes are increased and public debt starts its convergence to its SS level. Consumption decreases a bit more than in FRB in the period after the shock, due to the increase of taxes. Once again under robustness with this tough countercyclical fiscal policy the convergence process is oscillant and lasts more than with FRB.

Under *FRP* public debt decreases and taxes increase more than in FRB the period of the shock. The initial drop in consumption is higher under *FRP*, and the initial recession is also amplified by this fiscal policy. As in flexible rates, investment decrease less and the fall of output in  $t=1$  is lower. In this second period public debt increases and taxes are decreased. Consumption in period 1 is higher under *FRP* due to the decrease of taxes and also due to the lower decrease of output. The convergence as happens with FRC combined with robustness is oscillant.

## 6 The dynamic effects of a financial shock under different regimes

In Table 1 we show the effects of the financial shock during the period of the shock and the next 9 periods under the different financial situations and policy regimes that we are considering. We show the sum of consumption losses, the standard deviation of consumption, and the same measures for output. We select 7 possible values of  $\alpha^G$ , the same 3 that we were showing previously ( $-0.4$ ,  $0$  and  $3.5$ ), and 4 new possible values ( $-0.6$ ,  $0.4$ ,  $1.4$  and  $3.5$ ). We add  $-0.6$  to have an idea of what could be the results of the strongest procyclical fiscal policy consistent with determinacy. We include  $0.4$  to have an idea of how the system behaves under a moderate countercyclical fiscal policy, and  $1.4$  to have an intermediate case. Finally we include  $\alpha^G = 8$  to have an even stronger countercyclical fiscal policy than the one we analysed earlier.

One thing that emerges clearly from the table is that for the same financial situation and for the same  $\alpha^G$ , the performance in the four criteria that we show is always better under flexible exchange rates (with the only exception of a very strong countercyclical fiscal policy under robustness,  $\alpha^G = 8$ ). Fixed exchange rates contributes to stabilize the dollarized public debt and for the same financial situation and cyclicity of fiscal policy, taxes paid after the financial shock are lower under fixed exchange rate (not shown in the table). This has a beneficial effect on consumption but it is more than compensated by the negative output effects associated to the shock under fixed rates.

A moderate countercyclical fiscal policy ( $\alpha^G = 0.4$ ), implies less consumption volatility compared with a neutral fiscal policy or a procyclical fiscal policy no matter the financial situation or the exchange rate regime. The benefits of this kind of fiscal policy in terms of the other criteria that we are showing in the table is less clear when we are in a robust financial situation or under flexible rates.

A strong countercyclical fiscal policy seems to be a good policy (in terms of the criteria that we are focusing) when we are in a vulnerable situation, and particularly if the vulnerable situation is combined with fixed exchange rates. In this latter case a very strong countercyclical fiscal policy ( $\alpha^G = 8$ ) is better in terms of all the criteria than other fiscal policy. One possible interpretation of this fact is that the higher the financial frictions and the nominal rigidity the greater the possible stabilizing role of a countercyclical fiscal policy.

Finally the procyclical fiscal policy implies in all the cases more consumption volatility than the neutral fiscal policy, but can have some benefits in terms of lower output and consumption losses, especially when we are in a robust and/or flexible rates environment.

Table 1: Effects on consumption and output of a financial shock

Flexible exchange rates								
$\alpha^G$	Financial robustness				Financial vulnerability			
	$\Sigma c$	$\sigma_c$	$\Sigma y$	$\sigma_y$	$\Sigma c$	$\sigma_c$	$\Sigma y$	$\sigma_y$
-0.6	-0.14	1.42	-0.15	0.18	-1.77	3.07	-0.35	0.43
-0.4	-0.65	0.48	-0.27	0.05	-2.18	1.47	-0.48	0.17
0	-0.75	0.17	-0.34	0.07	-2.40	0.64	-0.59	0.15
0.4	-0.83	0.14	-0.39	0.09	-2.59	0.45	-0.66	0.17
1.4	-1.01	0.16	-0.48	0.10	-2.98	0.31	-0.76	0.16
3.5	-1.26	0.23	-0.74	0.13	-2.94	0.23	-0.78	0.14
8	-1.15	0.56	-2.26	0.24	-2.06	0.22	-0.68	0.12

Fixed exchange rates								
$\alpha^G$	Financial robustness				Financial vulnerability			
	$\Sigma c$	$\sigma_c$	$\Sigma y$	$\sigma_y$	$\Sigma c$	$\sigma_c$	$\Sigma y$	$\sigma_y$
-0.6	-0.45	1.48	-0.75	0.24	-2.97	3.11	-2.22	0.63
-0.4	-0.98	0.53	-0.81	0.14	-3.07	1.64	-2.02	0.44
0	-1.02	0.22	-0.77	0.12	-3.02	0.81	-1.68	0.34
0.4	-1.04	0.17	-0.73	0.12	-3.06	0.56	-1.48	0.28
1.4	-1.10	0.18	-0.61	0.13	-3.31	0.33	-1.28	0.20
3.5	-1.82	0.50	-0.89	0.22	-3.11	0.21	-1.09	0.14
8	-6.54	6.94	-10.95	2.51	-2.11	0.21	-0.84	0.12

## 7 Conclusions

Working with an open economy model with sticky wages, where debt is dollarized, and risk premium is determined by domestic net worth, (Céspedes et al., 2004) show that flexible exchange rates dominate fixed ER and play insulating role against external shocks.

The intuition of such result is the following: an external shock requires a real devaluation, that can be reached by two ways: nominal depreciation (flexible exchange rates) or deflation (fixed exchange rates). In both cases we have negative balance sheet effects, but with fixed exchange rates (and nominal rigidity) we have an additional recessive effect. Monetary policy can do nothing against financial frictions but can undo the effects of nominal rigidity.

We include a public sector that collects taxes, and issues public bonds in international markets in foreign currency. The government uses these resources to pay debt issued in previous period, and can follow three different fiscal rules: i) Countercyclical fiscal rule (*FRC*); ii) Procyclical fiscal rule (*FRP*) or iii) Neutral fiscal rule (*FRB*).

As in (Céspedes et al., 2004) we distinguish a situation of financial vulnerability from a situation of financial robustness.

In our framework flexible exchange rates dominate fixed exchange rates as in (Céspedes et al., 2004).

The conclusion about the desirability of flexible rates is not affected by the importance of financial imperfections. Nevertheless the convergence to the steady state after a shock is easier in a financially robust economy than in a vulnerable one.

A countercyclical fiscal policy diminishes the probability of being in a vulnerable situation. In our benchmark case, depreciation after a financial shock is lower in the FRC case with

respect to  $FR\bar{B}$ . The investment decrease is higher under financial robustness and lower under financial vulnerability (for  $\alpha^G$  large enough).

In the FRP case depreciation after a financial shock is higher compared with the situation under  $FR\bar{B}$ , while the investment decrease is lower.

The benefits of a counter cyclical fiscal policy tends to be maximum when financial vulnerability is combined with fixed exchange rate. In this case FRC can diminish the initial recession (in fixed exchange rates depreciation is associated with recession) and (for  $\alpha^G$  large enough) also the posterior drop in the level of activity (due to the capital decrease).

The system seems to be more stable under FRC and financial robustness (we have determinacy for more values of  $\alpha^G$  in this situation). On the contrary under a strong pro-cyclical fiscal policy the equilibrium is not determined.

In a flexible exchange rates environment, the benefits of a counter cyclical fiscal policy is less clear. In this situation a pro cyclical fiscal policy will increase depreciation (that is not too harmful in this context) but investment decrease (and the following decrease in production) will be lower.

Fiscal policy can alleviate financial frictions and can also undo the effects of nominal rigidity when monetary policy can not (fixed exchange rates).

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## A determinacy of equilibrium. The case of two predetermined variables and one non predetermined<sup>12</sup>

Given the following rational-expectations model:

$$\begin{bmatrix} E_t z_{t+1} \\ x_{t+1} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + a e_t$$

where  $z_t$  is a single non predetermined endogenous state variables,  $x_t$  is a 2-vector of predetermined endogenous state variables,  $e_t$  is a vector of exogenous disturbances terms, and  $A$  is a 3x3 matrix of coefficients. Under this conditions, rational-expectations equilibrium is determinate if and only if the matrix  $A$  has exactly two eigenvalues inside the unit circle (i.e.  $|\varphi| < 1$ ), and the other outside the unit circle.

Considering the following characteristic equation of matrix  $A$ :

$$P(\varphi) \equiv \varphi^3 + A_2 \varphi^2 + A_1 \varphi + A_0 = 0$$

This equation has two roots inside the unit circle and one outside if and only if:

either (Case I)

$$P(1) = 1 + A_2 + A_1 + A_0 > 0 \quad (\text{A.1})$$

and

$$P(-1) = -1 + A_2 - A_1 + A_0 > 0 \quad (\text{A.2})$$

and

$$A_0^2 - A_0 A_2 + A_1 - 1 < 0 \quad (\text{A.3})$$

or (Case II)

$$P(1) = 1 + A_2 + A_1 + A_0 < 0 \quad (\text{A.4})$$

$$P(-1) = -1 + A_2 - A_1 + A_0 < 0 \quad (\text{A.5})$$

and condition A.3 holds

or (Case III) conditions A.1 and A.2 holds and

$$|A_0| < 1 \quad (\text{A.6})$$

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<sup>12</sup>The proof developed in this section is similar to the proof of Proposition C.2, Appendix C of (Woodford, 2003), where the case of two non predetermined variables and one predetermined is analyzed.

or (Case IV) conditions A.4, A.5 and A.6 holds

*Proof:*

*Case I:* We have  $P(1) > 0$ ,  $P(-1) > 0$ ,  $P(\lambda) < 0$ , when  $\lambda \rightarrow -\infty$  and  $P(\lambda) > 0$ , when  $\lambda \rightarrow \infty$ . By continuity there are an even number of real roots between -1 and 1 (0 or 2 real roots), and an odd number of real roots (1 or 3 real roots) lower than -1 and an even number of real roots greater than 1 (0 or 2 real roots).

We have to discard the case of 0 real roots between -1 and 1. Condition A.3 can also be written as:

$$(\lambda_1\lambda_2 - 1)(\lambda_1\lambda_3 - 1)(\lambda_2\lambda_3 - 1) < 0 \quad (\text{A.7})$$

In order to have 0 real roots between -1 and 1, we need: a) 3 real roots lower than -1 or: b) 1 real root lower than -1 and two real roots greater than 1. In both cases it is easy to see that A.7 does not hold. This implies that we have two real roots inside the unit circle and one outside.

*Case II:* The proof is similar to case I. By continuity there are an even number of real roots lower than -1 (0 or 2 real roots), an even number of real roots between -1 and 1 (0 or 2 real roots), and an odd number of real roots greater than 1 (1 or 3 real roots). Again we can discard the case of 0 real roots between -1 and 1, using condition A.3, so we have determinacy also in this case.

The proof of cases III and IV are also similar. We also have to discard the case of 0 real roots between -1 and 1 and condition A.6 can be written as:

$$|-\lambda_1\lambda_2\lambda_3| < 1 \quad (\text{A.8})$$

But for A.8 to hold we need at least one  $|\lambda| < 1$  so we have two real roots inside the unit circle and one outside.

## B Corporate risk premium and Steady State

The problem that generates a corporate risk premium in our model is the same as in (Céspedes et al., 2004), that in turn is taken from (Bernanke et al., 1999).

Entrepreneurs finance their investment using their net worth ( $P_t N_t$ ) and borrowing from foreign lenders in dollars in international markets. Entrepreneurs and lenders are risk neutral.

Credit market frictions determines the appearance of an external finance premium.

Investment generates  $\omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$  next period, where  $\omega$  is a random shock with a known distribution.  $\omega$  has a mean value of one, it is observed freely by the entrepreneur but lender has to pay a monitor cost of  $\zeta \omega_{t+1}^j K_{t+1}^j (R_{t+1}/S_{t+1})$ , in order to observe it. If the realization of  $\omega$  is low and entrepreneur can not pay what was stipulated ( $\omega < \bar{\omega}$ ) then the lender pays the monitor costs and all the product goes to the lender.

In these conditions (Céspedes et al., 2004) show that:

$$1 + \eta_{t+1} = \frac{R_{t+1} S_t}{Q_t S_{t+1} (1 + \rho_{t+1})} \quad (\text{B.1})$$

where  $\eta$  is the external finance premium (risk premium), that depends on  $\bar{\omega}$

$$1 + \eta_{t+1} = \Delta(\bar{\omega}) \quad (\text{B.2})$$

Monitoring costs of equation 12 are given by:

$$\zeta \int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j) \equiv \Phi(1 + \eta_{t+1}) \quad (\text{B.3})$$

The only difference between our case and the one of (Céspedes et al., 2004) is that in our model we have a public sector. Public expenditure is part of aggregate demand which can affect product level (and net worth) or investment level. Workers have to pay taxes, and so consumption and aggregate demand is decreased.

The proof of the existence of a non-stochastic steady state is similar to the one of Appendix B of (Céspedes et al., 2004). They show, following (Bernanke et al., 1999), that provided that  $0 < \delta(1 + \rho) < 1$ , there exists a unique positive steady state solution of  $\bar{\omega}$ , that pins down the values of  $\eta$  and  $\frac{QK}{N}$ . The steady state values of the other variables can be obtained using the following equations:

$$Y = AK^\alpha \quad (\text{B.4})$$

$$Q = S^{1-\gamma} \quad (\text{B.5})$$

$$\frac{\alpha Y}{QK} = (1 + \rho)(1 + \eta) \quad (\text{B.6})$$

$$N + SD = QK \quad (\text{B.7})$$

$$Y = \gamma[(1 - \alpha)Y + QK - T] + SX \quad (\text{B.8})$$

$$B = \bar{B} \quad (\text{B.9})$$

$$T = \frac{BSi^G}{(1+i^G)} \quad (\text{B.10})$$

$$1+i^G = (1+\rho)(1+\eta^G) \quad (\text{B.11})$$

$$\eta^G = \overline{\eta^G} \quad (\text{B.12})$$

Combining equations 10 and 3, and linearizing we arrive to:

$${}_{t-1}y_t - (q_t - p_t) - k_{t+1} = \rho'_{t+1} + \eta'_{t+1} + {}_t(s_{t+1} - p_{t+1}) - (s_t - p_t) \quad (\text{B.13})$$

Linearizing equation 11, combining with the linearized version of 12, and the linearized version of 9 and with equations B.2, B.3, B.13 and B.6 and operating we arrive to:

$$\begin{aligned} \eta'_{t+1} = & \phi^1 \eta'_t + \mu(q_t - p_t + k_{t+1} - y_t + \\ & \mu\delta(1+\rho)\psi[(s_t - p_t) - {}_{t-1}(s_t - p_t) - (y_t - {}_{t-1}y_t)] + \end{aligned} \quad (\text{B.14})$$

where  $\phi = \delta(1+\rho)(1-\psi\mu) + \mu[\delta(1+\rho)(1+\psi)\eta + \delta(1+\rho) - 1](\frac{v}{\varepsilon_\Delta})$ . We defined  $v$  and  $\varepsilon_\Delta$  in the same way as (Céspedes et al., 2004).<sup>13</sup>

Now, combining equation 28 with equation B.13 we obtain equation 33, and using again equation 28 combined with B.14 we have 21.

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<sup>13</sup> $v$  is the elasticity of  $\int_0^{\bar{\omega}} \omega_{t+1}^j dH(\omega_{t+1}^j)$  and  $\varepsilon_\Delta$  is the elasticity of  $\Delta$  in B.2, both with respect to  $\bar{\omega}$

## C Steady State. determinacy of equilibrium

In the  $FR\bar{B}$  case the the dynamics of the model can be summarized by equations 21 and 22 (that are simplified given that  $\alpha^G = 0$ ). Assuming no shocks ( $\rho'_{t+1} = \eta'_{t+1} = 0$ ), these equations can be written in state space form:<sup>14</sup>

$$\begin{bmatrix} z_{t+1} \\ \eta'_{t+1} \end{bmatrix} = \begin{bmatrix} \lambda_k^{-1} + \mu \left( \frac{1-\lambda_k}{\lambda_k} \right) & \phi \\ \mu \left( \frac{1-\lambda_k}{\lambda_k} \right) & \phi \end{bmatrix} \begin{bmatrix} z_t \\ \eta'_t \end{bmatrix}$$

Eigenvalues are real if:  $[(1 + \mu)(1 - \lambda_k) + (1 + \phi)\lambda_k]^2 > 4\lambda_k\phi$ .

A sufficient conditions for the two eigenvalues to be on opposite sides of the unit circle is:  $\phi < 1 + \mu$

Finally we compute the saddle path:  $z_t = \xi \eta'_t$ , where  $\xi = \frac{\lambda_k}{\mu(1-\lambda_k)}(\varphi_2 - \phi)$ , where  $\varphi_2$  is the smaller eigenvalue. We can show that  $\varphi_2 - \phi$  is negative and so  $\xi$  is negative too.

In the  $FRC$  and  $FRP$  case, i.e. when  $\alpha^G \neq 0$ , the dynamics of the model can be summarized by equations 21, 22 and 23. Assuming no shocks ( $\rho'_{t+1} = \eta'_{t+1} = 0$ ), these equations can be written in state space form in the following way:

$$\begin{bmatrix} z'_{t+1} \\ \eta'_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -I\mu & \phi & J \\ -(1+\mu)I & \phi & J+H \end{bmatrix} \begin{bmatrix} z'_t \\ \eta'_t \\ z_t \end{bmatrix}$$

where  $I = \frac{PY}{QK}\alpha^G$ ;  $J = \frac{\mu(1-\lambda_k)}{\lambda_k} + \frac{\mu I}{1+i^G}$  and  $H = \frac{1}{\lambda_k} + \frac{I}{1+i^G}$ .<sup>15</sup>

The characteristic equation of the matrix of coefficients is the following:

$$P(\varphi) \equiv \varphi^3 + (-J - H - \phi)\varphi^2 + (H\phi + (1 + \mu)I)\varphi - \phi I = 0 \quad (C.1)$$

In order to see determinacy we have to check the conditions that we described on Appendix A.

Under a countercyclical fiscal policy ( $FRC$ ,  $\alpha^G > 0$ ) it is easy to see that  $P(-1) < 0$ , so equation A.2 is not satisfied and we can discard cases I and III.

In order to have cases II or IV, equation A.4 has to hold. This implies:

$$(1 + \mu - \phi) \frac{Ii^G}{1 + i^G} + [\phi - (1 + \mu)] \frac{1 - \lambda_k}{\lambda_k} < 0 \quad (C.2)$$

With a neutral fiscal policy ( $\alpha^G = 0$ ), this condition collapses to  $\phi < 1 + \mu$ , the same condition that we see at the beginning of this appendix. Assuming  $\phi < 1 + \mu$ , and with  $\alpha^G > 0$  in order to fulfill condition C.2 we need  $i^G$  and/or  $\alpha^G$  not too big. Intuitively if we have an extreme counter cyclical fiscal policy ( $\alpha^G$  too big), then we need  $i^G$  small, otherwise after an adverse shock we will increase too much debt and if the interest rate that have to pay public debt is high we can derive in an unsustainable public debt dynamics. In a sense fiscal policy can be so counter cyclical as public debt interest rate allows.

If C.2 is satisfied we need an additional condition (A.3 or A.6) to have a determinate equilibrium under a counter cyclical fiscal policy. We concentrate here on condition A.6 (case IV of appendix A). Under  $FRC$ , this condition implies:

<sup>14</sup>This problem is formally identically to the one analysed by (Céspedes et al., 2004).

<sup>15</sup>If  $\alpha^G = 0$  we can show that the previous system collapses to the system presented at the beginning of this section.

$$\alpha^G \phi < \frac{QK}{PY} \quad (C.3)$$

Condition C.3 implies that  $\phi$  (persistence of corporate risk) and  $\alpha^G$  can not be both too big. Intuitively if we have an aggressive countercyclical fiscal policy, public debt can change too much after a shock which can affect the dynamics of corporate risk (see equation 21) and with a very persistent corporate risk process we can derive in an explosive dynamics.

Under a procyclical fiscal policy (FRP,  $\alpha^G < 0$ ) we also have  $P(1) < 0$ , and so equation A.1 does not hold and we discard cases I and III.

To obtain cases II or IV, equation A.5 has to hold. Under FRP this implies:

$$\alpha^G > \frac{-(1 + \phi) \left[ 1 + \frac{1}{\lambda_k} \right] + \mu \frac{1 - \lambda_k}{\lambda_k} \frac{QK}{PY}}{(1 + \mu + \phi) \left( 1 + \frac{1}{1 + i^G} \right)} \equiv \alpha^{G*} \quad (C.4)$$

Condition C.4 implies that there is a threshold of how pro-cyclical fiscal policy can be in order to ensure a determinate equilibrium,  $\alpha^G$  can not be too negative.

Finally we need an additional condition (A.3 or A.6) to have a determinate equilibrium. As in the FRC case we focus on condition A.6 (case IV of appendix A). Under FRP, this condition implies:

$$\alpha^G \phi > \frac{QK}{-PY} \quad (C.5)$$

As happens under FRC, condition C.5 implies that  $\phi$  (persistence of corporate risk) and  $\alpha^G$  (in negative) can not be both too big.

We compute the saddle path:  $z_t = \xi_z z_{t-1} + \xi_\eta \eta_t'$ . The expressions for  $\xi_z$  and  $\xi_\eta$  are given by equations C.6 and C.7, respectively.

$$\xi_z = - \frac{(I\mu - J\varphi_3)(\phi - \varphi_2)\varphi_2 - (\phi - \varphi_3)\varphi_3(I\mu - J\varphi_2)}{(I\mu - J\phi)(\varphi_2 - \varphi_3)} \quad (C.6)$$

$$\xi_\eta = \frac{(\phi - \varphi_2)(\phi - \varphi_3)}{I\mu - J\phi} \quad (C.7)$$

Where  $\varphi_3$  is the smaller eigenvalue, and  $\varphi_2$  is the intermediate one.

According to calibration exercises,  $\xi_z$ , and  $\xi_\eta$  increases when  $\alpha^G$  increases.  $\xi_z$  starts from approximately  $-1$  at the most procyclical fiscal policy compatible with determinacy is zero in  $FR\bar{B}$  and it is approximately 1 at the most countercyclical fiscal policy compatible with determinacy.  $\xi_\eta$  is always negative but it is a big coefficient (in absolute value) at the most procyclical fiscal policy compatible with determinacy and it is close to zero in a strong countercyclical fiscal policy.